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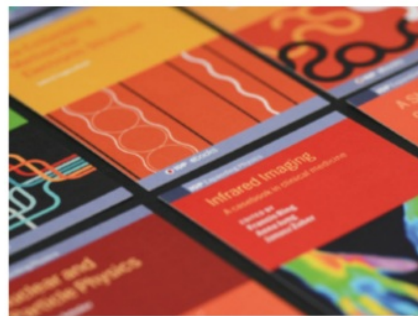
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The ability of students' visual thinking in solving integral problems

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Abstract. Problem-solving is a high-level mental activity visualization has been an area of interest in a number of researchers concern with mathematics education. *Visual thinking* is an important part of mathematical thinking. The purpose of this study will describe the ability of students' visual thinking in solving integral problems. This research uses a descriptive qualitative method by using purposive sampling technique. The results of this study show that there are three levels of visual ability. The first, the student in a *non-visual* is unable to representing and interpreting problems (concepts) graphically, however, be able algebraically but incomplete. The Second, the student in a *local-visual* is able to generate specific information on diagrams, however unable to drawing and using diagrams in problem-solving. The third, the student in *global-visuals* able to understand algebra and geometry as an alternative language and they indicate complete competence in problem-solving.

1. Introduction

Learning is a holistic process in which all aspects of knowledge are developed and constructed together. Through comprehensive learning, students can have the sensitivity to learning things in different situations. As learning progresses, schematic networks are formed, added, or modified as needed [1]. Thus, students must constantly update and develop their knowledge by adjusting the existing network of schemas with the new information they receive. Naturally, in mathematical learning, we communicate mathematical ideas through different forms, such as words, images, symbols, objects, or actions. The submission of ideas of mathematics can be manifested in various forms, which are more studied in the context of representation. Representation can be a tool to reinforce, reason and communicate an idea the term representation refers to the process as well as the result. [2]. There is no direct access to mathematical objects but only to their representation [3].

Visualization has an important role in learning. Visualization involves both external and internal representations (or images), one can define visualization as processes involved in constructing and transforming both visual images and all of the representations of a spatial nature that may be used in drawing figures or constructing or manipulating them with pencil and paper [4].

Thinking is a mental activity. Thinking is a process of generating mental representation by transforming information [5,6,7]. In addition to the transformation of information that thought involves



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the activity of manipulating information on memory [8]. There are three central theories: 1) a double-coded theory that can be represented in two visual and verbal codes; 2) the conceptual theory that visual information and verbal information are represented in terms of abstract propositions; 3) The functional equality theory that non-verbal images systems and symbolic-verbal systems involve similar processes. The processing of visual information on the mind is called visual thinking [7].

Several factors influence the visual thinking of the individual. That not everyone also masters the creation and manipulation of mental imagery because it is influenced by individual factors, definitions or tasks, experiences, or interactions [6]. Another explanation that functional functionality, mental stability, and the addition of a perceptual framework can be a mental barrier to visual thinking [5]. Basically, the role of visual thinking is very important to solving integral problems, which is one of the materials in the calculation, which focuses on the visual elements[9].

Visual thinking is used to solve problems. Problem-solving always covers all aspects of human activity, be it in the fields of science, law, education, business, sports, health, industry, literature, etc.[6]. One of the objectives of the mathematics lesson according to Permendiknas no. 22 on content standards is for troubleshooting. Some problems are more easily solved visually [10].

Some experts have studied problem-solving. The stages of problem-solving are the formation of problematic probations, the planning of the most probable solutions, the reformulation of the subject, the implementation and the evaluation of the results [11] and the most commonly used in problem-solving is to understand, plan, execute and re-check [12].

The problem of this study is the integral problem which is a mathematical problem of high level. Cognitive development to solve a high level or formal math problems, through the visual platonic (using visual images according to the idea of plato) and digital symbolic (using numbers or symbols digital) [13]. In addition to mastering mathematics, mathematics students must be able to communicate with mathematics. The ability of mathematical communication includes the ability to explain the concepts and facts of mathematics in the form of simple, easy, logical, clarify and identify principles that do all the work. The characteristics of the students, among others, bring a lot of experience, have the initiative, and independently. Characteristics of other student learning behavior, that is: goal, having a set of life experiences, goal-oriented and relevant, tend to be practical and require rewards [14].

There are several indicators of visual thinking ability: a) To understand algebra and geometry as alternative languages; b) To extract specific information on diagrams; c) To represent and interpret problem (or concept) graphically; d) To draw and use diagrams as an aid in problem-solving; e) To understand mathematical transformations visually [15]. The purpose of this research is to describe students' level of visual thinking ability in solving integral problems.

3 Experimental method

This research is a descriptive qualitative research. The subjects of this study were students of the Department of Tadris Mathematics IAIN Tulungagung semester V academic year 2017/2018, which had 35 students. Sampling technique using purposive sampling. The subjects used must: a) has taken calculus courses; b) has IP 3.3. The main instrument of this study is the researchers themselves. Data collection techniques using tests and interviews. The test is used to determine students' ability to think visually in solving integral problems. The questionnaire comprised two problems indefinite integral. Each interview lasted about 40-50 minutes and was video and audio-taped. The analysis is performed on valid data. Steps in data analysis include categorization or classification of data, data reduction, data exposure, interpretation of data, and drawing conclusions.

3. Result and discussion

Students' visual thinking skills is derived from data or information provided by students to solve integral problems. All information processing activities in integral problem-solving are not expressed by the subject. Therefore, research in an effective search is reactive, adaptive, holistic and conscious in an indescribable context. The test instrument in this study consists of 2 problems, which are shown in figure 1 below:

1. Use algebra and geometry to calculate $\int_{-5}^5 |x+3| dx$.
2. Determine the area of the shaded area in the following figure by using the integral

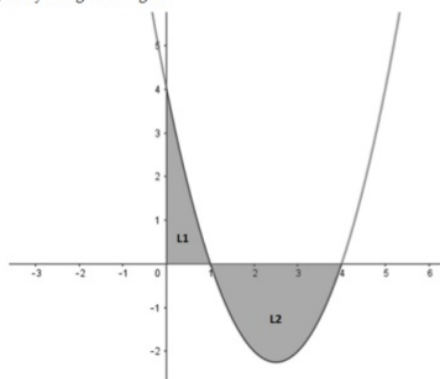


Figure 1. The study questionnaire.

Figure 1 shows that the given problem is to solve algebra in graphs and graphs in algebra. The following will describe students' visual thinking ability to solve the integral problem of each level.

3.1. Student's visual thinking ability at a non-visual level

Figure 2 shows that the subject of Diana Kumalasari (DK) seems unable to understand the meaning of $|x+3|$ where DK immediately writes $\int_{-5}^5 |x+3| dx = \left| \frac{1}{2}x^2 + 3x \right|_{-5}^5 dx$, from here it appears that DK is unable to use symbolic representations with perfection, unable to understand mathematics using numeric numbers/symbols. You cannot describe the functions provided correctly. DK is also unable to finish using the graph, but in the answer, there are graphs but do not understand the graphs created. The subject could solve the problem by substituting it. Here is the subject's answer.

$$\begin{aligned}
 \int_{-5}^5 |x+3| dx &= \left| \frac{1}{2}x^2 + 3x \right|_{-5}^5 \\
 &= \left| \frac{1}{2} \cdot (5)^2 + 3 \cdot 5 \right| - \left| \frac{1}{2} \cdot (-5)^2 + 3 \cdot (-5) \right| \\
 &= \left| \frac{1}{2} \cdot 25 + 15 \right| - \left| \frac{1}{2} \cdot 25 - 15 \right| \\
 &= \left| \frac{25 + 30}{2} \right| - \left| \frac{25 - 30}{2} \right| \\
 &= \frac{55}{2} - \frac{5}{2} \\
 &= \frac{50}{2} = 25
 \end{aligned}$$

Geometri:

Figure 2. Diana's answer for task 1.

By solving the two topics, we see that the DK responds only by redrawing the given problem. The subject does not include the area under and above the x-axis at all, not able to create the functional equation of the given graph. The subject is not able to calculate the surface by looking at the image. Basically, the subject in the non-visual group uses only a symbolic representation to solve the given problem. This indicates that the students consider symbolic representation as a support tool. Additionally, the students in this group were inclined to rely on analytical thinking instead of visual thinking. This leads to a tendency to be cognitively fixed on standard figures and procedures instead of recognizing the advantages of visualizing the tasks. Shows that visualization can be an obstacle to solving mathematical problems, especially when the mental image of a particular subject controls students' thinking. In this group, the mental image of the subject underlies them by creating an image to solve the problem[4].

3.2. Student's visual thinking ability at a local visual level

From the results given, the subject of Sri Wahyuni (SW) is shown in Figure 3 that SW is unable to understand the meaning of $|x + 3|$, where SW wrote directly $\int_{-5}^5 |x + 3| dx = \left[\frac{1}{2}x^2 + 3x \right]_{-5}^5$, from which it emerges that SW is able to use symbolic representations but not yet perfect, able to understand mathematics using numeric numbers/symbols but not yet perfect. And not able to manipulate the area using a graphical representation by changing the integral symbol. Here is the subject's answer.

$$\begin{aligned}
 \int_{-5}^5 |x+3| dx &= \left[\frac{1}{2}x^2 + 3x \right]_{-5}^5 \\
 \Rightarrow \int_{-5}^5 |x+3| dx &= \left[\frac{1}{2}x^2 + 3x \right]_{-5}^5 \\
 &= \left[\frac{1}{2}(5)^2 + 3(5) \right] - \left[\frac{1}{2}(-5)^2 + 3(-5) \right] \\
 &= \left[\frac{25}{2} + 15 \right] - \left[\frac{25}{2} - 15 \right] \\
 &= \left| \frac{25+30}{2} \right| - \left| \frac{25-30}{2} \right| \\
 &= \left| \frac{55}{2} \right| - \left| \frac{-5}{2} \right| = \frac{55}{2} - \frac{5}{2} = \frac{50}{2} = 25 \text{ satuan luas}
 \end{aligned}$$

Figure 3. Sri's answer for task 1.

Figure 4 shows that the subject SW seem to have understood the meaning of the graph above the x-axis and below the x-axis, although it is still not perfect. already able to create the functional equation of the given graph. First, try to make the equation $y_p = a(x_1 - p)(x_2 - q)$ then enter the points, so that the value of $a = 1$, then substituted for the equation so that the equation $y_p = 1(x_1 - 1)(x_2 - 4)$ procedurally seen the subject is very understanding but conceptually even less. Can also be seen Subject able to calculate the area by looking at the picture as a whole but not perfect.

$$\begin{aligned}
 & y = a(x-p)(x-q) \\
 & -2 = a(3-1)(3-4) \\
 & -2 = a(2)(-1) \\
 & -2 = a(-2) \\
 & a = 1 \\
 & \Rightarrow y = 1(x-1)(x-4) \\
 & y = x^2 - 4x - x + 4 \\
 & y = x^2 - 5x + 4 \\
 & \int_0^1 x^2 - 5x + 4 \, dx = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \right]_0^1 \\
 & = \left[\frac{1}{3}(1)^3 - \frac{5}{2}(1)^2 + 4(1) \right] - \left[\frac{1}{3}(0)^3 - \frac{5}{2}(0)^2 + 4(0) \right] \\
 & = \frac{1}{3} - \frac{5}{2} + 4 = \frac{2 - 15 + 24}{6} = \frac{11}{6} \\
 & \int_1^4 x^2 - 5x + 4 \, dx = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \right]_1^4 \\
 & \Rightarrow * = \left[\frac{64}{3} - 40 + 16 \right] - \left[\frac{1}{3} - \frac{5}{2} + 4 \right] \\
 & = \left[\frac{64 - 120 + 48}{3} \right] - \left(\frac{11}{6} \right) \\
 & = \frac{-8}{3} - \frac{11}{6} = \frac{(-16) - 11}{6} = -\frac{27}{6} \\
 & \text{Luas} = L_1 + L_2 \\
 & = \frac{4}{6} + \left(-\frac{27}{6} \right) = \left| \frac{-16}{6} \right| = \frac{16}{6} \text{ satuan luas.} \\
 & \text{∴ Luas daerah } L_1 \text{ dan } L_2 = \frac{16}{6} \text{ satuan luas.}
 \end{aligned}$$

Figure 4. Sris' answer for task 2.

Generally, from the results show in figure 4, that the students in this group have begun to coordinate the representation systems of definite integrals. These students can perform representation transformations in separate representation systems. The students in this group differ from those at the non-visual level in that they have understood algebra and geometry as alternative languages for the concept of definite integral, and developed visual methods to see mathematical concepts and problems better. The students in this group used visual representations as induced image and they induced the visual images mainly from the analytic thinking. Although their visual thinking favors local rather than global thinking, this restricted visualization effectively blocks the accomplishment of their tasks. In addition, the selected visuals reflect only one aspect of the integral concept, which has important implications for their ability to perform other tasks. An important element of visual thinking is the ability to recognize that algebraic responses are based on geometry; interviews show that this component does not exist in the minds of many students [9].

3.3. Student's visual thinking ability at a global visual level

Figure 5 shows that the subject of Siti Lailatul (SL) shows that SL is able to understand the meaning of $|x + 3|$, where SL directly looks for points to make the curve of his function, from which it appears that SL is able to use symbolic representations to perfection, using numeric numbers/symbols. And able to manipulate the area using the graphical representation by changing the integral symbol, and able to complete with a graphic representation to perfection. Here is the subject's answer.

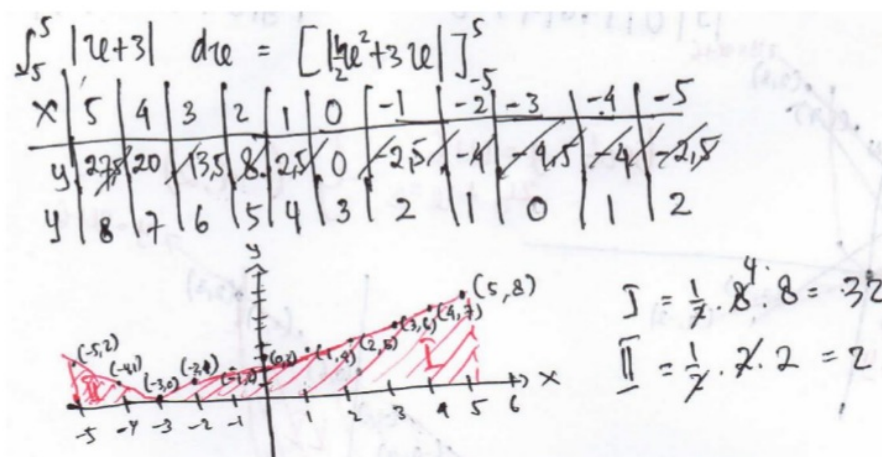


Figure 5. Siti's answer for task 1.

Figure 6 shows that from the results given the SL subject seems to have understood the meaning of the graph above the x-axis and below the x-axis, with perfection. already able to create the functional equation of the given graph. First try to write the bounds of the point (1,0), (4,0) make the equation of the curve of squares $(x - x_1)(x - x_2)$ across 2 points, then enter the points, so that $f(x) = (x - 1)(x - 4)$ given the subject is very understanding of the procedural and conceptual level.

Handwritten work for task 2. At the top, the points $(0, 4)$ and $(1, 0)$ are noted. The functional equation is derived as $f(u) = (u-1)(u-4) = u^2 - 5u + 4$. The area under the curve is calculated as $\int_0^4 f(u) du = \int_0^4 (u^2 - 5u + 4) du = [\frac{1}{3}u^3 - \frac{5}{2}u^2 + 4u]_0^4 = [\frac{1}{3} \cdot 64 - \frac{5}{2} \cdot 16 + 16] - 0 = [\frac{64}{3} - 40 + 16] = [\frac{64}{3} - 24] = \frac{8}{3}$. The final result is $3\frac{2}{3} + 1\frac{5}{6} = 4\frac{14}{6} = 6\frac{2}{3} = 6\frac{1}{2}$.

Figure 6. Siti's answer for task 2.

In general, on figure 6 the subject is in this group imagination image are different from memory images to students, because imagination images did not really exist in the past, but were generated through the students' creative process. The added accuracy in this group of students' drawings could be characterized as *introducing suitable notation* [13] which was Polya's recommendation when using visual representations [12]. For Siti, a visualization *is a powerful tool* for exploring *mathematical problems* and for attributing meaning *to* the concept of integral and relationship between them.

4. Conclusion

The results of the data analysis show that the main obstacles prevent the students from moving freely into representational systems for certain integral concepts that they lack the ability to visualize abstract relationships and information. nonfigurative in representation and visual imagery. The development of visualization capabilities, which can affect the relationship between graphical representation and other representations, improves integrated problem-solving performance. First, students in non-visual cannot represent and interpret the problem (concept) graphically, however, into algebra but not complete. Second, students in local visual can generate specific information on the diagram and cannot draw and use the diagram in problem-solving. Third, global-visual students are able to understand algebra and geometry as an alternative language.

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References

- [1] Van de Walle J, Karp KS, Bay-Williams and Jennifer M 2013 *Elementary and Middle School Mathematics: Teaching Developmentally (8th edition)* (USA: Pearson Education Inc)
- [2] NCTM 2000 *Principles and Standards for School Mathematics* (Virginia: NCTM)
- [3] Duval R 1999 Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking Basic Issues for Learning *Twenty-First Annu. Meet. North Am Chapter Int. Gr Psychol Math Educ.* **25**(1) pp 3–26
- [4] Presmeg N 2006 Research on visualization in learning and teaching mathematics *Handb Res Psychol Math Educ.* pp 205–235
- [5] Suharnan 2005 *Psikologi Kognitif* (Surabaya: Srikandi)
- [6] Stenberg R J and Karin S 2012 *Cognitive Psychology* (USA)
- [7] Solso R L, Maclin O H and Maclin M K 2007 *Psikologi Kognitif 8ed Alih Bahasa Mikael Rahardanto dan Kristianto Batuadji Editor Wibi Hardani* (Jakarta: Erlangga)
- [8] Santrock J W 2009 *Educational Psychology Edisi 3 Buku 1* (Jakarta: Salemba)
- [9] Zimmermann W 1991 *Visual Thinking in Calculus* In: Zimmermann W and Cunningham S (Eds) *Visualization in Teaching and Learning Mathematics* (Washington DC: Mathematical Association of America)
- [10] Depdiknas 2006 *Kurikulum Tingkat Satuan Pendidikan* (Jakarta: Depdiknas)
- [11] Glass A L and Holyoak K J 1986 *Cognition (2nd ed.)* (Singapore: McGraw-Hill Book Company)
- [12] Polya G 1973 *A New Aspect of Mathematical Method Second edition* (Princeton New Jersey: Princeton University Press)
- [13] Tall 1995 Cognitive Development, Representations and Proof This paper was prepared for the *Conference on Justifying and Proving in School. Mathematics, Institute of Education* pp. 27–38.
- [14] Alfeld P 2000 *Understanding Mathematics a Study Guide Department of Mathematics* College of Science University of Utah
- [15] Chih-Hsien Huang 2013 Engineering students' visual thinking of the concept of definite integral *Global Journal of Engineering Education* **15**(2)

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