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The Reconstruction of Improper Fraction Concept through Analogy Problems in Students of Prospective Elementary School Teacher

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Abstract. The concept of improper fraction has not been comprehensively understood by the students of prospective elementary school teacher so that it is necessary to reconstruct the concept. This research aims at analyzing the reconstruction of the improper fraction concept using analogy problem. The research is exploratory qualitative research. Conceptual reconstruction is independently designed through a task in the form of 3 questions that can guide students to reconstruct the concept independently. The instruments used in this research are 3 test questions for 44 students and in-depth interviews on 3 selected research subjects. The result of the research shows that 1) there are 27 students who could reconstruct the concept independently in a perfect manner that are called the independent type, 2) there are 15 students who were able to reconstruct the concept but only on a simple problem that are called the dependent type 3) there are 2 students who did not succeed in reconstructing the concept that are called the stagnant type. This research concludes that the reconstruction of the concept can be done without teaching the concept, but instead by providing problem solving tasks that can be solved independently. The future researchers also need to conduct research on developing students' self confidence in solving problems independently.

INTRODUCTION

The understanding of mathematics will be used to solve problems in life [1]. Good learning needs to involve teaching concepts, facts, principles, and procedures in a balanced way. However, according to Lunde there are three mathematical knowledge that must be mastered by students such as conceptual, procedural, and declarative knowledge [2]. Mathematical knowledge can be in the form of concepts, procedures, or declaratives. In practice, good learning requires a balance of conceptual and procedural understanding where procedural skills should be based in a correct conceptual knowledge [3]. Learning should not only memorize procedures but also provide opportunities for students to think [1]. Therefore, procedural knowledge should be understood first before proceeding to the mastery of other knowledge so that the material being studied becomes meaningful knowledge for the students. Concepts should be mastered well before learning the procedure.

The concept should be properly constructed by the students. It is because the concepts that have not been understood result in the failure of students in producing solutions [4]. Conceptual knowledge focuses on a deep

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understanding of meaning including the relationship between operations [5]. Conceptual knowledge is easier for students to understand through informal learning, involving object observation, the use of pictures or manipulative models, and appropriate exercises. Concept learning is an important part of learning that needs to be considered by both teachers and students. Ideally, learning does not only emphasize the mastery of procedures.

Learning process in Indonesia is dominated by procedural teaching. The habit of students being taught and learning to use procedural rules has an impact on students' abilities in terms of conceptual and problem solving [1], [3]. Learning process that does not emphasize understanding results in less meaningful learning that leads to fragmentation of students understanding [1]. Students' failure in learning mathematics occurs because of the difficulties in understanding and abstracting concepts, as well as relating concepts to everyday life [1]. In fact, the connection between existing concepts and new concepts needs to a concern for teachers [6]. Therefore, a comprehensive concept mastery by the student needs to be pursued so that it can be achieved through the correct construction of the concept.

Ideally, the concept is correctly constructed by the student. The concepts that have not been understood results in students' failure in producing solutions [4]. The failure to construct concepts begins with students' thinking errors in constructing mathematical concepts include: correct & incorrect pseudo thinking, analogy thinking, concept placing, and logical thinking [6]. On the other hand, the forms of mathematical concept construction errors are (1) pseudo construction, (2)construction holes, (3) mis-analogical construction, and (4) mis-logical construction [7]. Misconceptions can also lead to failure in constructing concepts [8] so the concept construction process should be done correctly and designed with care.

Mumu, & Charitas state that based on constructivism theory, the concept construction process must be carried out actively by students through individual interactions with their environment [8]. Presentation of the concept can be done in different ways on the same concept [5] . The process of concept construction can be achieved well when the process occurs actively within the students by utilizing the situation or environment around the students. This results in correct concept that is formed within the students. The process is also undergone meaningful learning so that it can be used in the application of procedures and in the time when studying the next material. This leads to the fact that the students do not experience misconceptions that result in inadequate understanding of the studied meaning when they are learning.

Based on the researchers' teaching experience for 15 years at Elementary Teacher Education Study Program of Islamic State University Sayyid Ali Rahmatullah Tulungagung, it shows that there are several concepts that are not well understood by students. One of the concepts that students do not fully understand is the concept of improper fraction even though their procedural understanding is good. This will also make an inadequate relational understanding of concepts, procedures, principles, and facts in solving problems. This condition is found in most students that come from almost all provinces in Indonesia. This indicates that there is a similar method in teaching mathematics in Indonesia which emphasizes procedural understanding more.

The result of preliminary study conducted by researchers on January 5, 2021 shows that most of the students had not been able to understand the concept of improper fraction. This condition is supported by the fact where out of 44 students given one question about the concept of improper fractions with moderate difficulty, only 6 students (14%) were able to understand the concept of improper fraction. Meanwhile, 38 students (86%) could not understand the concept of improper fraction. This is a concern because they will become teachers who will teach the concept of improper fraction in the future. If they have not been able to understand the concept, it can be predicted that they will also fail in teaching improper fraction concept later. As stated by Subanji, failure in learning mathematics will have a bad impact because it will result in failure to understand the next material [1].

Previous research has also not revealed much about teaching improper fraction concepts to students. Other research tends to focus on other fractional materials [9]–[13] Hackenberg, et.all conducted research on the relationship between multiplication of fractions and the ability to write equations[9] while Loc.et.all conducted research on understanding concept, addition, subtraction and multiplication of students in Vietnam [13]. Meanwhile, Mendiburo, et.all conducted research on the development of learning tools in the form of the HALF program [10]. In addition, Jayanthi, et.all conducted research on the study of factors that influence success in studying fractions[11]. It appears that the concept of improper fractions has not been widely explored by previous researchers. They tend to focus on fraction learning process on other materials because there are many problems in fraction material.

Fraction is a difficult material for students[10], [12]. This is reasonable because there is a complex problem with fraction[12]. One form of the complexity of the problem with fraction is that the results of the operation can produce values that are not commonly found in integer operations[13]. The result of such operation is an unreasonable value for children [12]. Meanwhile, according to Loc. et.all, if students complete the multiplication operation by multiplying the numerator by numerator and denominator by denominator, it is a natural thing to happen to elementary school students [13]. Therefore, it is not uncommon for students to have difficulty in understanding fraction material and teachers need to be aware of it. Awareness of this condition can lead teachers to make good learning plans for fraction material so that it can be implemented in fraction learning process.

Theoretical studies and field facts show that concept of improper fraction is not well understood by students including the college students. This can be triggered by the condition that mathematics in higher education is more abstract than mathematics at previous levels [8]. The concept that has not been properly constructed at the previous level of education leads to the importance of effective and efficient concept construction design. Concept construction is more effective when the concept is presented in the form of problem solving followed by clear work instructions[14]. If the students fail in constructing the concept, it is necessary reconstruct the concept. Mumu, & Charitas state that concept reconstruction can be used to overcome misconceptions [8].

Concept reconstruction requires good planning by tracking the knowledge that the student already has and the knowledge that student does not yet have [5], [8]. Concept reconstruction can be performed by giving analogy problems [8]. It is because the use of analogies helps the students find new concepts through previously known knowledge [15]. The use of analogy can broaden the understanding of concepts and help fix misconceptions experienced by students [16]. Reconstruction is carried out to students who have studied the concept but the concept has not been understood correctly so it needs to be reconstructed which leads to a correct concept possessed by the students.

One way to reconstruct the concept is to provide an analogy problem to the student. The use of analogy in reconstructing the concept of improper fraction can be done using the Cocburn & Littler three steps framework. The first, second, and third steps are carried out by giving series of tasks such as opening task, connecting task and target task [17]. The form of the problem refers to the theory developed by Remigio, Yangco & Espinosa which categorize the format of analogy problem into two types such as visual (non verbal) and verbal [16]. Meanwhile, Stenberg states that the reviewed analogy problem can be classified into verbal and geometric analogy [18]. The problem used in this research is verbal analogy problem.

Reconstruction of improper fraction concepts in this research is designed using problems which will guide students to reinvent concepts independently. It is because the reconstruction of concept in adults can be done independently if the task instructions are clear [19]. Therefore, a problem with clear task instructions is designed in the form of an analogy problem. This enables the reconstruction of improper fraction concept being performed by students through solving analogy problems in the form of verbal problems which contain three components such as the opening task, connecting task, and target task. Reconstruction of improper fraction concept for students can be carried out by giving three analogy problems with low, medium, and high difficulty levels. The problems can lead students to reconstruct the improper fraction concept that they possess. The type of response produced by students can occur in 3 types such as independent, dependent, and stagnant type. The independent type can solve the three problems correctly. They can answer correctly without the need of external intervention. Dependent type can solve problems correctly on low and medium difficulty problems. This type still requires teacher's intervention to make them understand difficult concept. While the stagnant type is unable to solve the problem given in this research. This type cannot reconstruct the concept independently. They need deep intervention to be able to reconstruct the concept that they have. The way how the characteristics of each type in reconstructing the improper fraction concept are needs to be explored through research entitled "Reconstruction of Improper Fraction Concept through Analogy Problems in Students of Prospective Elementary School Teacher".

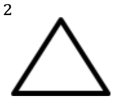
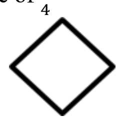
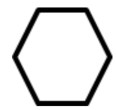
METHODOLOGY

This research is exploratory qualitative research. There were 44 students who participated in this study. These students are from Elementary Teacher Education Study Program of Islamic State University Sayyid Ali Rahmatullah Tulungagung. Three subjects are selected in this research such as students who were able to correctly answer all three questions, students who were able to correctly answer one or two questions, and students who cannot answer correctly on all three questions. Each group of students is taken one subject. The selected subjects were interviewed about the process of reconstructing improper fraction concept when solving the problems. The question problems in this research are designed to be able to independently reconstruct improper fraction concept for students.

The test questions are problems about improper fraction in the form of pictures which contain analogies. The analogy can be seen from the sub-questions presented, starting from the opening task, connecting task, and target task.

The problems are designed from low, medium, and high difficulty problem in a row with starting from number 1, 2, and 3. The form of the problem can be seen in Table 1 below.

TABLE 1. Research Problems

Problem Structure	**Analogy Problem**		
	1 (Low difficulty problem: Opening Task)	**2 (Medium difficulty problem: Connecting Task)**	**3 (High difficulty problem: Target Task)**
Given	The following shape has a value of $\frac{1}{2}$ 	The following shape has a value of $\frac{1}{4}$ 	The following shape has a value of $\frac{1}{2}$ 
A Shape with the Value of 1	Draw a shape with the value of 1 based on the first image.	Draw a shape with the value of 1 based on the first image.	Draw a shape with the value of 1 based on the first image.
Searched Image	Based on the first and second shape, draw a shape with a value of $\frac{5}{2}$	Based on the first and second shape, draw a shape with a value of $\frac{3}{2}$!	Based on the first and second shape, draw a shape with a value of $\frac{5}{3}$!

If students are able to correctly answer questions number 1 to 3, they are categorized as the students who were able to reconstruct independently up to the high-level difficulty concept so that they belong to the independent type. The students who were able to answer correctly questions number 1 and 2 are included in the category of students who are able to reconstruct concepts in a dependent manner. It is because in order to understand difficult concepts, they still need intervention from the teacher so that they enter the dependent type. Meanwhile, students who couldn't correctly answer all questions are categorized as the ones who are unable to reconstruct concepts through analogy problems. These students are included in the stagnant type. It is because they are not at all able to find concepts even on the easy-level difficulty problems.

The procedure in this research went through these steps namely 1) the students were given 3 questions in three consecutive weeks where at each meeting they were given one question within 15 minutes; 2) the students completed the questions given by writing answers on a piece of paper; 3) the results produced by the students are categorized; 4) interviews were conducted for each type of students to find out the process of finding ideas; 5) the results of the interviews were compared with the solutions produced; 6) the descriptions of the reconstructing process of improper fraction concept in students are discovered.

FINDING

The results show that out of 44 students, it is discovered that 1) there were 27students who could reconstruct the correct concept independently (independent type); 2) there were 15 students who were able to reconstruct the concept but only on a simple problem (dependent type); 3) there were 2 students who did not succeed in reconstructing the concept (stagnant type). The process of concept reconstruction for each type is described in the following explanation. The independent, dependent, and stagnant reconstruction types are given the initials RI, RD, and RS.

Reconstruction of Improper Fraction Concept on IR (Independent Reconstruction)

The test result shows that IR was able to correctly answer the three questions given. The answers produced by IR show that whether on low, medium, and high level difficulty questions, the subject is able to reconstruct concepts through analogies. In the opening, connecting, target task, the subject managed to solve it well. Interviews with IR show that the subject was able to reconstruct the concept independently. In the opening task such as in question number 1, the subject could solve it correctly. The subject thought that to draw $\frac{5}{2}$ if the known shape had a value of $\frac{1}{2}$ by identifying the shape for 1 1, then $\frac{5}{2}$ was broken down as a multiplication of $5\, x\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. This summation was grouped so that the result was 2 added by $\frac{1}{2}$. Since the subject found the shape for 1 in the previous step, this subject could easily describe $\frac{5}{2}$ as $2\frac{1}{2}$. The following interview result shows the process of concept reconstruction experienced by IR.

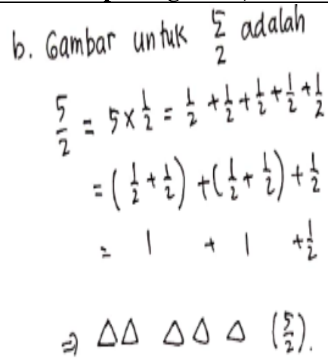
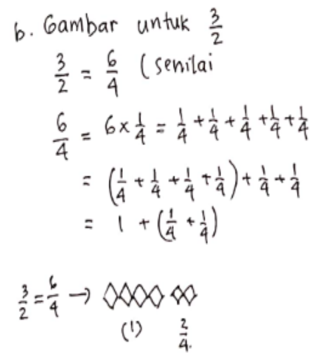
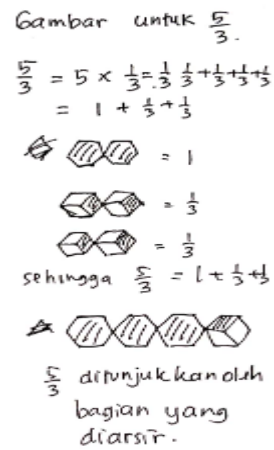
> " At first I was confused. It is because it was at that time, I faced a question like that. Then I tried to think about it again and I found out that there were other ways that I could use. If there was already a shape for $\frac{1}{2}$ and the one that I should find was $\frac{5}{2}$, then I tried to find the result from 1 initially like what was being asked from the question. After I got 1, I changed $\frac{5}{2}$ into a multiplication of $5\, x\frac{1}{2}$ and then I did repetitive summation. From that point I found out that the result was 2 added by $\frac{1}{2}$. Because shape 1 was already there, I only need to draw $1 + 1 + \frac{1}{2}$ with a shape of 5 triangles."

On the second question, IR also answered correctly in a similar way to question number 1. However, there were times when the subject was a little confused. It is because what is known is $\frac{1}{4}$ but what the subject is looking for is $\frac{3}{2}$ then the first step the subject did was to look for shape for 1 first. After that, the subject thought about converting $\frac{3}{2}$ into a fraction equals to the denominator 4. The subject found out that $\frac{3}{2} = \frac{6}{4}$. With the same steps as in question number 1, the subject could find the correct answer as the solution. The following interview excerpt shows the process experienced by IR.

> " At first I thought that it was question number 1 but then I looked at it again and it turned out as harder than the previous one. It is because the one that I had to find and the one that I knew had different denominators. Then I tried to think about the answer. I remembered that in fraction, there was a term equal fraction and I used that to convert $\frac{3}{2}$ into $\frac{6}{4}$ since what was known from the question was the denominator which was 4. However, I looked for the value for 1 beforehand. When I discovered this idea, I continued using solution steps similar to question number 1."

TABLE 2. IR Test Result

Problem Structure	Analogy Problem: 1 (Low difficulty problem: Opening Task)	Analogy Problem: 2 (Medium difficulty problem: Connecting Task)	Analogy Problem: 3 (High difficulty problem: Target Task)
Given	Diketahui : $\frac{1}{2} \to$ △	Diketahui : $\frac{1}{4} \to$ ◇	Diketahui : $\frac{1}{2} \to$ ⬡
A Shape with the Value of 1	a. Gambar untuk 1 adalah $1 = 2 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$ △△ $\to 1$	a. Gambar untuk 1 adalah $1 = 4 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ ◇◇◇◇ $\to 1.$	Gambar untuk $1 = 2 \times \frac{1}{2}$ $1 = \frac{1}{2} + \frac{1}{2}$ ⬡⬡ $\to 1.$

Problem Structure	Analogy Problem		
	1 (Low difficulty problem: Opening Task)	2 (Medium difficulty problem: Connecting Task)	3 (High difficulty problem: Target Task)
Searched Image			

The third problem could be solved correctly by IR. On this question, the subject thought over and over again to be able to answer the question. The subject tried to describe the shape with a value of 1 first. After that, the subject thought of a way to answer. As in the previous answer, the subject tried to present $\frac{5}{3}$ as a multiplication of $5\ x\frac{1}{3}$ and broke it down into repetitive summation $5\ x\frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. Next, the subject grouped the summation into $\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right) + \frac{1}{3} + \frac{1}{3}$, the part in brackets is 1 and it has been described in the previous step. In order to describe the shape of a third, the subject first drew 1, which was two hexagons. In each of the hexagons, the subject divided it into three equal parts and shaded one part so that one third could be illustrated as two parts which were shaded. The same way is done to find the value of the second third. Then the shaded parts were collected. A third of 2 could be formed from four shaded parts while a hexagon had 3 equal parts. Therefore, $2\ x\frac{1}{3}$ could be illustrated in terms of one hexagon and one shaded part of the hexagon which is divided into three equal parts. The final shape was 3 hexagons and a third of the hexagon. The following interview excerpt explains the meaning of IR's answer.

> "The third question made me feel confused. It is because between the one that is known and being asked seem unrelated. Then I tried to draw 1 first in hopes that I could find something that I could use in answering this question. I was right. After I find the shape of 1, I thought about converting $\frac{5}{3}$ into a multiplication of $5\ x\ \frac{1}{3}$ and I continued the step as a repetitive summation of $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. On this part, 3 times triangles equal to 1 so that $\frac{5}{3}$ could be broken down into 1 added by two thirds. I had discovered the shape of 1 before, I just needed to draw the shape of a third. I went back to the fraction concept that a third is a part of a whole that includes three equal parts. Then I drew a third by drawing two hexagon shapes, then I divided each of the hexagon into three and I selected one part. Therefore, a third can be illustrated as 2 parts of the shaded part. I did the same step to find the other triangle. So that, $\frac{2}{3}$ can be illustrated as 4 shaded parts. One hexagon contains 3 shaded parts, then 4 shaded parts equal to 1 hexagon added by 1 shaded part. The final shape is the answer that three hexagons is added by a third of a hexagon."

The result of the tests and interviews show that IR is able to correctly reconstruct concepts from low to high level difficulty problems. The subject is able to perform it independently and does not require intervention from the teacher. Reconstruction of improper fraction concept can be carried out through the problem given in this research. RI's understanding and reasoning seems to be in a good quality. The subject can use the knowledge possessed and relate to that knowledge to solve the problem at hand. IR is very persistent in trying to find answers and carefully analyze the ideas so that it produces the correct answer.

Reconstruction of Improper Fraction Concept on DR (Dependent Reconstruction)

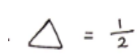
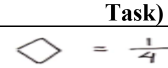
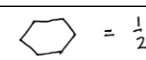
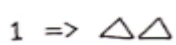
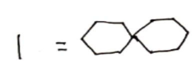
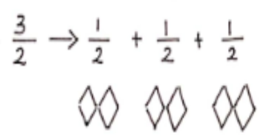
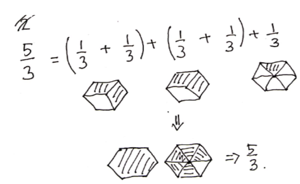
The result of RD's test shows that this subject was able to solve moderately on low level difficulty problem. As for high level difficulty problem, the subject had not been able to solve. The interview result confirms this condition. It shows that for question number 1, DR was able to solve the problem in this research by stating $\frac{5}{2}$ as repetitive summation so that there were 5 halves and could be drawn with 5 triangles. The following interview excerpt demonstrates this.

> "I solved this question by drawing 1 first. It can be illustrated as two triangles. On the question that requires us to draw $\frac{5}{2}$, I drew five halves. It is because a half was represented by one triangle then $\frac{5}{2}$ could be illustrated as five triangles."

The second question was answered correctly by DR. This subject drew 1 first. In question number 1, the subject did not feel the need to draw 1 because the subject did not see any function to answer. However, in question number 2, the subject felt that the shape of 1 could help him think about the answer to question number 2. When what is known is a quarter and it is illustrated by a rhombus, then in order to find $\frac{3}{2}$, the subject added half three times which had the same meaning as one added by a half. The subject thought that half was twice a quarter. Therefore, half could be described as two rhombuses. It is because what is being asked is $\frac{3}{2}$, then 6 rhombuses were drawn. This result was shown from the result of following interview excerpt.

> "I looked for the shape of 1 like in question number 1. At first, I didn't realize that it could help me find the answer to this question. When I know the shape of 1, I could draw the value that was not yet discovered. $\frac{3}{2}$ could be described as $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$. A half and a half addition equals to 1. I could draw this by the 1 that I found in the previous step which were four rhombuses. Then I thought about a shape for a half. It is because a half is twice a quester then I drew a half with two rhombuses. At this point, I discovered 6 rhombuses."

TABLE 3. DR Test Result

Problem Structure	Analogy Problem: 1 (Low difficulty problem: Opening Task)	2 (Medium difficulty problem: Connecting Task)	3 (High difficulty problem: Target Task)
Given			
A Shape with the Value of 1			
Searched Image			

In the question number 3, DR had not managed to answer correctly. An error was made when describing a third. The subject didn't realize that one hexagon on the problem was worth half. The subject thought that a hexagon was worth one third so the resulting shape was wrong. After completing the answer, DR realized that there was something wrong with the answer. However, the subject was unable to fix the answer. The following is an excerpt of an interview session with DR.

> "I think the question number 3 is a bit hard. It is because what is known and what is asked seem to be unrelated. However, I tried to find the shape that is worth 1. After that, I looked for the shape for $\frac{5}{3}$ by adding the triangles 5 times. I drew the third with one shaded part from three parts of one hexagon. Therefore, there were five shaded parts and when those five shaded parts were collected, one whole hexagon and five parts of a hexagon

that were divided into six. Actually, I felt that there was something wrong with this answer after I finished writing down the answer. However, I could not fix it."

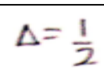
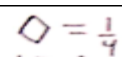
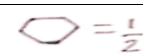
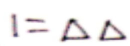
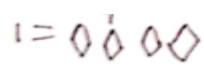
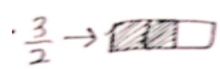
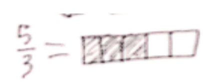
The result of the test and interview shows that the DR type students were able to solve the questions correctly up to the medium level difficulty question problem. For the high level difficulty question problem, the subject had not been able to complete them perfectly. It takes a little of teacher's intervention to be able to make DR solve the difficult problem. The intervention was meant so that the question given in this research can help DR reconstruct the concept that this subject has. However, the process was not perfect because the teacher's intervention was still needed so that DR is categorized in the dependent type in reconstructing the concept.

Reconstruction of Improper Fraction Concept on SR (Stagnant Reconstruction)

The test result from RS shows that SR could not answer correctly on the three questions. The subject was only able to correctly draw the picture of 1. On the other hand, the improper fractions were drawn incorrectly. SR seemed to flip the numerator into the denominator when the subject found a fraction with a larger numerator value. The following interview result shows the thinking process experienced by SR.

"I think this question is weird because we are asked to draw a fraction that has larger numerator. I've never seen a fraction that has higher numerator so that I was confused in answering this question. Then I flip between the numerator and denominator so that I have lesser numerator."

TABLE 4. SR Test Result

Problem Structure	**Analogy Problem**		
	1 (Low difficulty problem: Opening Task)	**2 (Medium difficulty problem: Connecting Task)**	**3 (High difficulty problem: Target Task)**
Given			
A Shape with the Value of 1			
Searched Image			

SR did not seem to be able to reconstruct the concept of improper fraction through the given problem. The perception of SR about fraction is whether the numerator should be less than the denominator. The subject could not interpret fraction as division where in the concept of division in fraction $\frac{5}{3}$ can be interpreted as 5 divided by 3. The subject also could not express the fraction of $\frac{5}{3}$ as a form of 5 times summation of a third. This subject was not able to explain the knowledge possessed so that the reconstruction of concept through problems given in this research took place in a stagnant manner. Such type needs to get a great support in order to understand the concept well.

DISCUSSION

The preliminary data shows that most students have not been able to construct the concept of improper fraction correctly. It is known from the initial test result about the concept of improper fraction which can only be completed correctly by 6 students or 16% of total students. It becomes a matter of concern because their achievement is still very low. A learning process which only focuses on the mastery of procedural skills make the students vulnerable in terms of understanding the concept, Purnomo.et.all and Subanji state a similar notion that the habit of students being taught and learning using procedural rules has an impact on inadequate conceptual skills [3], [7]. Therefore, some efforts are needed to reconstruct the concept.

The concept reconstruction is attempted to be done by giving an analogy problem about the improper fraction concept in this research. The problem given in this research is problem solving followed by clear working instruction because the design of such problem according to Sinha & Kapur will make the students understand about the concept[14]. The problem given has a context that is commonly encountered by students where mathematical contextualization according to Jameson & Fusco can help adult learners increase efficiency and reduce anxiety[19]. The problem presented is an analogy problem which consists of an opening task, connecting task, and target task. In regards of the difficulty levels, the opening task, connecting task, and target task are each categorized as low, medium, and high level difficulty problem. The design of analogy problems refers to the theory put forward by Cocburn & Littler who states that a teaching using analogy problems can be designed in 3 consecutive forms such as opening, connecting and target problems[17].

The test result of 44 students shows that the answers produced by students can be categorized into three types of responses such as 1) the student who answers all the questions correctly (as many as 27 students); 2) the student who correctly answers one or two questions (as many as 15 students); 3) the student who answers incorrectly on all questions (as many as 2 students). The first group of students is the students who are able to reconstruct the concept of improper fraction independently and is called the Independent Reconstruction (IR). The students who are able to answer correctly on question number 1 or 2 and incorrectly on question number 3 are students who have progressed in understanding the concept of improper fraction. However, their understanding is not perfect because it still requires teacher's intervention to be able to understand complex concepts. These students are categorized into the Dependent Reconstruction type (DR). The third group of students is the students who are unable to answer all questions correctly. Students of this type are not able to reconstruct their concept despite being given problem that can lead them to reconstruct the concept of improper fraction. It seems that the independent reconstruction of concept through analogy problem has changed the understanding of students' concepts. Initially only 16% of students understand the concept, after the concept reconstruction, there are 95% of students who can understand the concept. Only 5% of students do not experience the progress.

Independent Reconstruction (IR) type students are able to solve the problem by correctly answering the three questions. IR first analyzed all possibilities before writing answers. IR is a type of analytical thinker as described according to Rofiki, Nusantara, Subanji, who state that the characteristic of analytical thinker is to try to analyze all possible answers by breaking down difficult problems into simpler and more reachable forms [20]. RI also tried to relate the problem in this research to the concept of fractions and multiplication operation on fractions. It appears that this type of student is able to link between the concepts and procedures they know so that good reasoning is formed. Mallet and Purnomo state that the concept and procedure presented in a balanced manner can build the reasoning inside the students [3], [21].

The concept of fraction, both fraction as part of a whole or fractions as a division of the numerator by the denominator, were all analyzed and seen for their suitability with the problem presented in this research. The students of IR type have an extensive knowledge and are able to use it according to the required context. They dare to think outside the box and to present ideas in their own way and not only imitate what the teacher does but also dare to explore. Even though they dare to think freely, the answers produced are the ones that can be reasoned and have the right value. This shows that IR type students have the following characteristics: 1) they have a good understanding; 2) they are thorough and careful; 3) they are open-minded; 4) they dare to think out of the box; 5) they are able to relate the knowledge possessed to solve problems; and 6) they are able to validate every idea found so as to find the right answer effectively. Their ability and courage enable them to reconstruct concepts and solve problems independently. Active student participation is able to develop the concepts that they possess [22]

Dependent Reconstruction (DR) students are able to answer correctly on question number 1 or number 1 and 2. However, they are unable to answer question number 3 correctly. This shows that they are able to reconstruct the concept but only on the low and medium level difficulty level of concept. They have not been able to reconstruct a high level difficulty concept. It is because they are unable to apply the concept of fraction to the third problem. They have adequate knowledge but they have not been able to use it perfectly. Bishop.et.all states that the initial knowledge that the students have can help them in learning the next material [23]. Their courage to think outside the box has not been maximized so that they still have a fear of being wrong to answer differently from what was exemplified by the teacher. However, the problem given in this research is enough to help the students develop their courage to answer in an unusual way. This type of students needs to be given support to increase the courage to think and relate the ideas that they have. Thus, RD type of students have the characteristic such as: 1) they have a good understanding; 2) they are thorough but not careful enough; 3) they are quite open-minded; 4) they are not brave enough to think outside the box; 5) their inadequate knowledge makes them unable to relate the concept when needed; 6) they validate their ideas that they found but are less than perfect.

Stagnant Reconstruction (SR) students are students who do not experience changes in their thought process even though they are given problems that allow them to change their thinking. Therefore, they are experiencing a stagnant state. They do not dare to present different ideas from the teacher. If there is no example from the teacher, they cannot solve the problem. They tend to just duplicate what has been learned previously. These students learn less meaningfully because they only remember what they learned without understanding it. Less meaningful learning process results in fragmentation of thinking in students where one form of the fragmentation is the learning process by memorizing procedures only [1]. The characteristics of this type of students are 1) their understanding is inadequate; 2) they are less thorough and careful; 3) their thinking is less open-minded; 4) they do not have the courage to think differently from the teacher; 5) their knowledge is separated from each other; 6) their discovered ideas are not validated.

The results of this research indicate that the reconstruction of concepts in adults can be done without being explained by the teacher. They can construct concepts independently through the problems given either on low, medium, high level difficulty problems. Therefore, the concept reconstruction can be performed without having to explain the concept. A concept can be built through problem solving, one which is an analogy problem so that the teachers need to design varied learning processes. Glassmeyer et all states that teachers have a big influence on students' learning process [24]. Good understanding of prospective teacher is crucial. It helps them to become a good teacher that this process is influenced by their previous learning experience [24]–[26]. Professional learning development can help students learn and understand the material.

CONCLUSION

Based on the results of this research, it can be concluded that mathematical concepts which have not been properly understood by students can be reconstructed in various ways. One way to reconstruct concept is by giving analogy problems that can lead students to find the concepts independently. The use of analogy problems has proven to have helped students reconstruct their concepts that the time before being given the analogy problem, the concept was only understood by 14% of total students in this research. After being given the analogy problems on the concept, the students' understanding increased to 95%. The results of this study also show that there are three types of students' responses in constructing concepts such as the independent reconstruction, dependent reconstruction, and stagnant reconstruction type. Things that affect the formation of existing type include: the understanding of the context of the problem, the balance conceptual and procedural understanding on related material, the courage to think out of the box, the ability to relate the knowledge possessed, and the validation of ideas both before answering and after writing down the answers.

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